Optimization of a Stiffened Square Panel Subjected to Compressive Edge Loads

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Abstract

A DETAILED study was performed of the weight optimization problem for a square plate with a single, eccentric, blade-type stiffener. The plate was subjected to a compressive end load, acting in a direction parallel to the stiffener. The behavior of the stiffener as an eccentrically attached plate was fully taken into account. All three design parameters were varied: plate thickness, stiffener thickness, and stiffener height. The simultaneous collapse configuration of the plate was also studied in relation to its optimum properties.

Two separate optimum configurations were obtained. One of these could be approximated by the fully buckled (simultaneous collapse) configuration. The second, which possessed the lowest weight, could not.

Contents

The investigation was carried out within the scope of linear buckling theory of elastic plates, but taking into account the changing in-plane stresses during the buckling process and the rotational restraint between plate and stiffener. The two field equations of the buckling problem for each of the three connected plates included both the classical biharmonic terms in the normal displacements w_i and stress functions F_i, and the longitudinal in-plane prebuckling stress terms as is usually done in linear buckling analysis. Boundary conditions along the external supports parallel to the stiffener were: simple support for the normal displacements w_i of the main plates; free edge for the normal displacement w_3 of the stiffener, and zero in-plane force resultants. The conditions at the loaded edges perpendicular to the stiffener were of the SS3 type (simple supports for the w_i; zero in-plane transverse displacements and incremental normal stress). The boundary conditions along the common attachment line were those of joint equilibrium and of continuity of displacements and slopes.

The solution of the system was obtained by developing all terms as Fourier sine series in the longitudinal direction and then solving the resulting systems of ordinary differential equations in the transverse directions. Various approximations and simplifications could be made, depending on the range of the geometrical variables. For a given geometry, the solution determined the eigenvalues of the system, the lowest of which was the buckling stress $\bar{p}_{cr} = p_{cr}/E$.

The loading and geometry were naturally grouped into the following nondimensional parameters: $\bar{P} = P/EL^2$ = loading parameter; $\delta = t_1 h/tL$ = area ratio (stiffener to plate); $\gamma = (h^3 t_1/t^3 L)(1-\nu^2)$ = inertia ratio; also $\bar{\gamma} = \gamma(\bar{P})^{2/3}$; and $W = (t/L)(1+\delta)$ = cross section; also $\bar{W} = W(\bar{P})^{-1/3}$ where: P is the applied edge load; L is the dimension of square plate; E is Young's modulus; ν is Poisson's ratio; h is the stiffener height, t is the plate thickness; and t_1 is the stiffener thickness.

In terms of these definitions, the optimization problem could be stated as follows: Given $\bar{P}$, determine $\delta;\gamma;W(\geqq 0)$ so that W be as small as possible, subject to the constraint: $\bar{P}/W \leqq \bar{p}_{cr}$ = smallest eigenvalue of the system defined above.

The buckling modes of the plate were of three types: 1) symmetric buckling with the stiffner displaced without rotation and with one longitudinal half-wave; 2) antisymmetric buckling of the main plate with two longitudinal half waves and the stiffener providing rotational support without appreciable distortion; and 3) buckling of the stiffener blade as a long plate elastically supported by the main plate. Modes 2 and 3 coupled into a single asymmetric mode by the requirement of slope continuity with a sharply marked and narrow region of "transfer of dominance" (Fig. 2).

The simultaneous collapse (s.c.) or fully buckled state for this structure is the case where all three modes occur simultaneously. This would be the usual engineering approach to minimum weight design with s.c. serving as the "optimality criterion." It is an exact point in the solution domain since cross-restraint is lost when plate and stiffener buckle simultaneously.

A detailed study of the stability boundaries near s.c. has demonstrated that if cross-restraint is omitted near, as well as at, s.c. then it indeed becomes an exact minimum design configuration. Details of proof involved a study of the shifts in the stability boundaries of the three modes with the geometrical variables (Fig. 1). Note that in this case the "transfer of dominance" region degenerates into a point.

The relevant inequalities were:

$$\bar{W} \leqq 0.35(1-\nu^2)\delta^4(1+\delta)^2\bar{\gamma}^{-2} \text{ – stiffener buckling}$$

$$\bar{W}^3 \geqq 0.076(1-\nu^2)(1+\delta)^2\theta^{-2}$$

with $\theta = 1$ for antisymmetric buckling and

$$\frac{\delta+1.4}{\delta+0.35}\gamma - 16\delta\theta^2 = 3.24\theta(\alpha_1^{-1}\tanh\alpha_1 - \alpha_2^{-1}\tan\alpha_2)^{-1}$$

$$\alpha_{1,2} = \pi/2\sqrt{4\theta \pm 1}$$

for symmetric buckling. The intersection of the three boundaries was the s.c. design configuration with:

$$t/L = 0.3851\left[\frac{\delta^2(\delta+1.40)}{(\delta+0.35)(16\delta+7.22)}\right]^{1/2}$$

$$\bar{P} = 2.145\delta^2(1+\delta)(t/L)$$

$$h/L = 0.4037\delta^{1/2}; \quad t_1/t = 2.477\delta^{1/2}$$

In many practical designs (e.g., riveted panels) it may be advantageous not to rely on the capability of the joint to transmit rotational restraint. For this model, s.c. is the true minimum design configuration of the structure.

A more exact analysis combined the stiffener and plate antisymmetric modes into a single asymmetric mode. A

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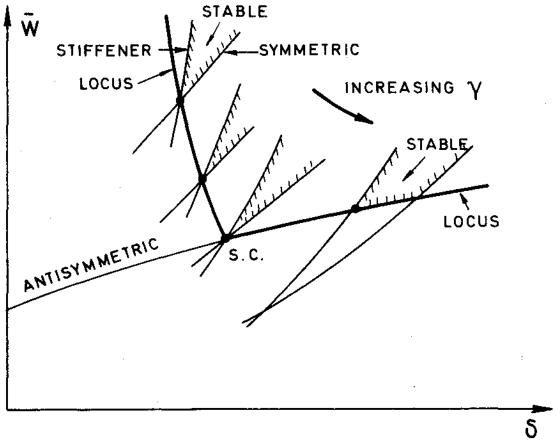

Fig. 1 Simultaneous collapse as an optimum design.

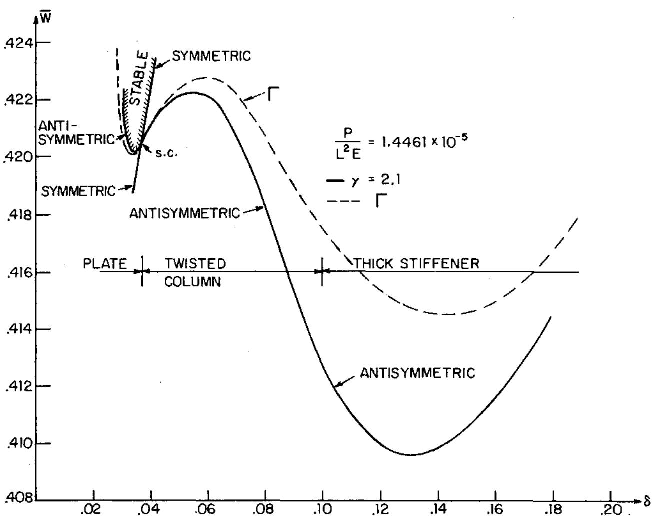

Fig. 2 Stability boundaries and locus.

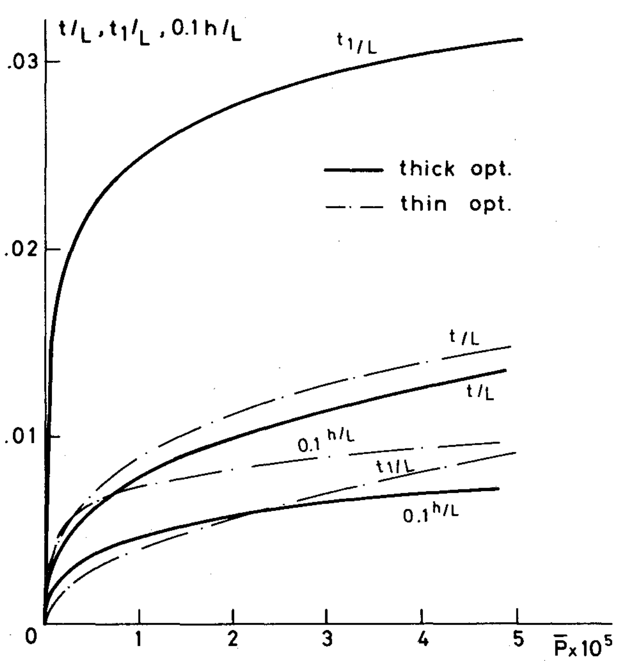

Fig. 3 Design variables at optimum locations.

typical asymmetric stability boundary is shown (for given $\bar{\gamma}$ and $\bar{P}$) in Fig. 2. The region to the left of s.c. is termed "stiffener plate region" where stiffener buckling is dominant and the plate provides rotational restraint. It has a minimum point somewhat to the left of s.c. and then the curve starts to climb to the shift into main-plate dominance, where material transfer from plate to stiffener is inefficient. In the region to the right of s.c., the stiffener provides rotational support to the plate mainly as an end loaded "twisted column." The maximum point reflects the rapidly increasing effectiveness of the stiffener as a rotational spring as it gets away from its buckling point. Eventually, a "thick stiffener" region is reached with a second upswing in the curve. The latter reflects the relative inefficiency of rotational springs near a clamped support.

For a given $\bar{\gamma}$ and $\bar{P}$, the intersection of the stable regions for the symmetric and asymmetric modes yields the stable region of the stiffened plate in the $\bar{W}$-δ plane (see example in Fig. 2). While it is not obvious from the example, a detailed analysis of the changes in the stability boundaries with δ and $\bar{\gamma}$ has led to the following rigorous conclusion: Let $\Gamma: \bar{W} - \bar{W}(\delta)$ be the locus of intersections of the two stability boundary curves with $\bar{\gamma}$ as a parameter, then the minimum points of Γ are optimum design configurations. The Γ curves for various $\bar{P}$ were then calculated by eliminating $\bar{\gamma}$ from the equations.

In Fig. 2 a typical Γ curve is shown. It is evident that two local minima exist—one in the "thin" stiffener region very near s.c. and one in the "thick" stiffener region. The latter is the absolute minimum weight design. Design curves for the two minima are given in Fig. 3. An analytical perturbation technique for the definitive location of the thin minimum was also devised.

Conclusions

1) The optimization problem for the stiffened square plate has two solutions; a thick (absolute) optimum, suitable for integral designs and a thin optimum which is more suitable for riveted connections since it is less demanding on the capability to transfer rotation.

2) The fully buckled design point is very close to the thin optimum. It becomes an exact minimum if, and only if, zero cross-restraint is postulated.

3) All three minima lie within a narrow region in the $\bar{W} - \bar{P}$ plane, with $\bar{W} \simeq 0.42$.

4) The minimum points are all located at intersection points of the (curved) uncoupled stability boundaries.

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